Basic relations of a unified theory of electrodynamics, quantum mechanics, and gravitation

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Quantum mechanics once started with Planck's energy-frequency relation which is derived here as an exemplification. This is done using the basic relations of a consistent scalar-vector-tensor theory founded on solid principles of classical physics including special and general relativity.

There is a direct way leading to a natural quantization of Maxwell's equations, to the Klein-Gordon equation with an unconventional normalization, or to the completion of Einstein's equations by a quantum energy-momentum tensor of matter.

Approximations yield the macroscopic phenomenological counterpart of the last implying the gravitational laws of motion, the classical Hamilton-Jacobi equation, or Schroedinger's conceptualizing eigenvalue problem. The theory seems open for extensions.

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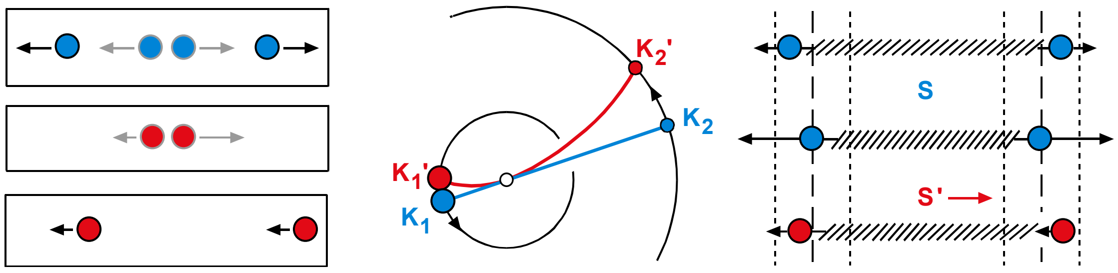
Dear colleagues, if the attribute 'classical' meant consistent, as it applies to Newton's mechanics, then the point is, that there had never been a classical electrodynamics so far. On the other hand consider a physicist 100 years ago, please. I like to show you how this fictive physicist - trying to overcome that logical incompleteness - might have found quantum mechanics as early as in the period between 1905 and 1915, when Einstein published his special and general relativity theory. As is well known, the breakthrough to the latter was obtained with the help of a generous friend, what is the reason that we can meet here in Marcel Grossmann's name.

I) The dynamic paradoxa of special relativity theory as prototypes of quantum mechanics

The energy-momentum postulate $$\int T_i^{\ 0} \mathrm{d}V = P_i \overset{!}{=} constant_i \big|_{\mathrm{SRT}} \qquad \text{(I,1)}$$

requires within SRT the conservation laws $$\partial_k T_i^{\ k} = 0 \qquad \text{(I,2)}$$

Relativity of simultaneity
in finite systems
with respect to S and S'
(strongly exaggerated)



1) Particles in a box 2) Rotator 3) Oscillator

Though flying temporarily in the same direction, both particles together with their respective carrier of interaction have to fulfill (I,2) with respect to S and S'. Since any objects are composed of charged particles, each process is subject to electrodynamics. Insertion into (I,2) of the

energy-momentum tensor $$T_{i\,(\text{conventional})}^{\ k} \equiv \left(-F_{il}F^{kl} + \tfrac{1}{4}\delta_i^k F_{lm}F^{lm}\right) + \mu_0 c^2 U_i U^k \qquad \text{(I,3)}$$

yields the equations of motion: $$F_{il}J^l = \mu_0 c^2 \frac{\mathrm{d}U_i}{\mathrm{d}\sigma}\,. \qquad \text{(I,4)}$$

To solve the paradoxa according to (I,2), the components of $T_i^{\ k}$ must fulfill the standard conditions of continuity and integrability.

II) The relation between current density and electromagnetic potentials as the key to a necessary extension of electrodynamics

Lorentz' ansatz for current density:
$$J^l = \rho_0 U^l \, . \qquad \text{(II,1)}$$

Definition:
$$F_{il} \equiv \partial_i \widetilde{A}_l - \partial_l \widetilde{A}_i \, . \qquad \text{(II,2)}$$

(II,1), (II,2) → (I,4):
$$\rho_0 U^l \left(\partial_i \widetilde{A}_l - \partial_l \widetilde{A}_i \right) = \mu_0 c^2 U^l \partial_l U_i \, . \qquad \text{(II,3)}$$

With the simplest ansatz
$$\rho_0 := e_0 \frac{\mu_0}{m_0} \equiv e_0 Q^2 \qquad \text{(II,4)}$$

the solution of (II,3) is
$$\widetilde{A}_i \overset{!}{=} -\frac{m_0 c^2}{e_0} U_i \, , \qquad \text{(II,5)}$$

because
$$U^l \partial_i U_l = \tfrac{1}{2} \partial_i \left(U^l U_l \right) = \tfrac{1}{2} \partial_i (1) = 0 \, . \qquad \text{(II,6)}$$

(II,5) → (II,1):
$$J^l = -\frac{e_0}{m_0 c^2} \rho_0 \widetilde{A}^l \, . \qquad \text{(II,7)}$$

Setting the extended potentials
$$\widetilde{A}_i \equiv A_i + \frac{c}{e_0} \partial_i S \, , \qquad \text{(II,8)}$$

we get the key:
$$\boxed{J^l = -\frac{e_0}{m_0 c} Q^2 \left(\partial^l S + \frac{e_0}{c} A^l \right)} \, . \qquad \text{(II,9)}$$

III) A first consistent variational principle of electrodynamics, quantum mechanics, and gravitation

The consistent variational principle
$$\delta\int\Big(\breve{\mathbf{\Phi}} + \sum_{\mathrm{K}} \overset{\frown}{\mathbf{\Phi}}_{\mathrm{K}} \Big)\mathrm{d}\Omega \;=\; 0 \qquad \text{(III,1)}$$

is composed of
$$\boxed{\overset{\frown}{\mathbf{\Phi}}_{\mathrm{K}} \;\equiv\; \tfrac{1}{4}\mathbf{f}_{\mathrm{K}}^{\,lm} f_{lm}^{\,\overline{\mathrm{K}}} - \frac{1}{2m_{\mathrm{K}}}\, q_{\mathrm{K}}^{2}\Big(\mathbf{s}_{\mathrm{K}}^{\,l} + \frac{e_{\mathrm{K}}}{c}\mathbf{a}_{\overline{\mathrm{K}}}^{\,l}\Big)\Big(s_{l}^{\,\mathrm{K}} + \frac{e_{\mathrm{K}}}{c} a_{l}^{\,\overline{\mathrm{K}}}\Big) + \tfrac{1}{2}\,\mathbf{m}_{\mathrm{K}} c^{2} q_{\mathrm{K}}^{2} - \frac{\hbar^{2}}{2m_{\mathrm{K}}}\mathbf{q}_{\mathrm{K}}^{\,l} q_{l}^{\,\mathrm{K}}}\,, \qquad \text{(III,2)}$$

where the index $\overline{\mathrm{K}}$ (non-K) is defined by
$$..^{\overline{\mathrm{K}}} \equiv \sum_{\nu \neq \mathrm{K}} ..^{(\nu)} \;, \qquad \text{(III,3)}$$

$$\overset{\frown}{\mathbf{\Phi}}_{\mathrm{ABC}...}\,(s_l, q, q_l, a_m, a_{lm}, g^{lm})$$
$$\equiv$$

or explicitly
$$\begin{aligned} &\tfrac{1}{4}\mathbf{f}_{\mathrm{A}}^{\,kl} f_{kl}^{\,\mathrm{BCD}...} - \frac{1}{2m_{\mathrm{A}}} q_{\mathrm{A}}^{2}\Big(\frac{e_{\mathrm{A}}}{c}\mathbf{a}_{\mathrm{BCD}...}^{\,l} + \mathbf{s}_{\mathrm{A}}^{\,l}\Big)\Big(\frac{e_{\mathrm{A}}}{c} a_{l}^{\,\mathrm{BCD}...} + s_{l}^{\,\mathrm{A}}\Big) + \tfrac{1}{2}\mathbf{m}_{\mathrm{A}} c^{2} q_{\mathrm{A}}^{2} - \frac{\hbar^{2}}{2m_{\mathrm{A}}}\mathbf{q}_{\mathrm{A}}^{\,l} q_{l}^{\,\mathrm{A}} \\ +&\tfrac{1}{4}\mathbf{f}_{\mathrm{B}}^{\,kl} f_{kl}^{\,\mathrm{ACD}...} - \frac{1}{2m_{\mathrm{B}}} q_{\mathrm{B}}^{2}\Big(\frac{e_{\mathrm{B}}}{c}\mathbf{a}_{\mathrm{ACD}...}^{\,l} + \mathbf{s}_{\mathrm{B}}^{\,l}\Big)\Big(\frac{e_{\mathrm{B}}}{c} a_{l}^{\,\mathrm{ACD}...} + s_{l}^{\,\mathrm{B}}\Big) + \tfrac{1}{2}\mathbf{m}_{\mathrm{B}} c^{2} q_{\mathrm{B}}^{2} - \frac{\hbar^{2}}{2m_{\mathrm{B}}}\mathbf{q}_{\mathrm{B}}^{\,l} q_{l}^{\,\mathrm{B}} \,, \\ &+ \ldots \end{aligned} \qquad \text{(III,4)}$$

and Einstein's
$$2\kappa\breve{\mathbf{\Phi}}(g^{lm}, g_{klm}) \;:=\; \mathbf{G} \;\equiv\; \sqrt{g}\, g^{um} g^{sv} g^{rw}\Big\{ \Gamma_{v,ur}\Gamma_{w,ms} - \Gamma_{v,um}\Gamma_{w,sr} \Big\}\,. \qquad \text{(III,5)}$$

III.1) The quantized Maxwell equations by variation with respect to the microscopic electromagnetic potentials a_i

$$\frac{\partial\widehat{\Phi}_{\mathrm{ABC}\ldots}}{\partial a_l^{\mathrm{A}}} = -\frac{1}{m_{\mathrm{B}}} q_{\mathrm{B}}^2 \left[\frac{e_{\mathrm{B}}{}^2}{c^2}\left(\mathbf{a}_{\mathrm{A}}^l + \mathbf{a}_{\mathrm{C}}^l + \ldots\right) + \frac{e_{\mathrm{B}}}{c}\mathbf{s}_{\mathrm{B}}^l\right] - \frac{1}{m_{\mathrm{C}}} q_{\mathrm{C}}^2 \left[\frac{e_{\mathrm{C}}{}^2}{c^2}\left(\mathbf{a}_{\mathrm{A}}^l + \mathbf{a}_{\mathrm{B}}^l + \ldots\right) + \frac{e_{\mathrm{C}}}{c}\mathbf{s}_{\mathrm{C}}^l\right] + \ldots$$

$$\frac{\partial\widehat{\Phi}_{\mathrm{ABC}\ldots}}{\partial a_l^{\mathrm{B}}} = -\frac{1}{m_{\mathrm{A}}} q_{\mathrm{A}}^2 \left[\frac{e_{\mathrm{A}}{}^2}{c^2}\left(\mathbf{a}_{\mathrm{B}}^l + \mathbf{a}_{\mathrm{C}}^l + \ldots\right) + \frac{e_{\mathrm{A}}}{c}\mathbf{s}_{\mathrm{A}}^l\right] - \frac{1}{m_{\mathrm{C}}} q_{\mathrm{C}}^2 \left[\frac{e_{\mathrm{C}}{}^2}{c^2}\left(\mathbf{a}_{\mathrm{A}}^l + \mathbf{a}_{\mathrm{B}}^l + \ldots\right) + \frac{e_{\mathrm{C}}}{c}\mathbf{s}_{\mathrm{C}}^l\right] + \ldots \qquad \text{(III,6)}$$

$$\ldots$$

$$\partial_k\left(\frac{\partial\widehat{\Phi}_{\mathrm{ABC}\ldots}}{\partial a_{kl}^{\mathrm{A}}}\right) = \partial_k\left[\mathbf{f}_{\mathrm{B}}^{kl} + \mathbf{f}_{\mathrm{C}}^{kl} + \ldots\right]$$

$$\partial_k\left(\frac{\partial\widehat{\Phi}_{\mathrm{ABC}\ldots}}{\partial a_{kl}^{\mathrm{B}}}\right) = \partial_k\left[\mathbf{f}_{\mathrm{A}}^{kl} + \mathbf{f}_{\mathrm{C}}^{kl} + \ldots\right] \qquad \text{(III,7)}$$

$$\ldots$$

1. Pair as an identity by definition: $$f_{ik;l}^{\mathrm{K}} + f_{kl;i}^{\mathrm{K}} + f_{li;k}^{\mathrm{K}} \equiv 0 \qquad \text{(III,8)}$$

2. Pair derived by variation: $$\boxed{\partial_k \mathbf{f}_{\mathrm{K}}^{kl} \equiv \tilde{\mathbf{j}}_{\mathrm{K}}^l = -\frac{e_{\mathrm{K}}}{m_{\mathrm{K}} c} q_{\mathrm{K}}^2\left(\mathbf{s}_{\mathrm{K}}^l + \frac{e_{\mathrm{K}}}{c}\mathbf{a}_{\overline{\mathrm{K}}}^l\right)} \; . \qquad \text{(III,9)}$$

$$\tilde{\mathbf{j}}_{\mathrm{K}}^l \equiv -\frac{e_{\mathrm{K}}}{m_{\mathrm{K}} c} q_{\mathrm{K}}^2 \tilde{\mathbf{s}}_{\mathrm{K}}^l \equiv -e_{\mathrm{K}} q_{\mathrm{K}}^2\left(\tfrac{1}{m_{\mathrm{K}} c}\mathbf{s}_{\mathrm{K}}^l + \frac{e_{\mathrm{K}}}{m_{\mathrm{K}} c^2}\mathbf{a}_{\overline{\mathrm{K}}}^l\right) \equiv e_{\mathrm{K}} q_{\mathrm{K}}^2\left(\mathbf{u}_{\mathrm{K}}^l + \mathbf{w}_{\overline{\mathrm{K}}}^l\right) \equiv \rho_{\mathrm{K}} \tilde{u}_{\mathrm{K}}^l \equiv (\tilde{\rho}_{\mathrm{K}}, \tfrac{1}{c}\vec{\tilde{\mathbf{j}}}_{\mathrm{K}}) \, . \qquad \text{(III,X)}$$

III.2) The covariant wave equation by variation with respect to the shape scalar *q*

$$\partial_l\left(\frac{\partial\hat{\mathbf{\Phi}}_{\mathrm{K}}}{\partial q_l^{\mathrm{K}}}\right) = \frac{\partial\hat{\mathbf{\Phi}}_{\mathrm{K}}}{\partial q^{\mathrm{K}}} \,. \tag{III,11}$$

$$\frac{\partial\hat{\mathbf{\Phi}}_{\mathrm{K}}}{\partial q^{\mathrm{K}}} = -\mathbf{q}_{\mathrm{K}}\left[\frac{1}{m_{\mathrm{K}}}\tilde{s}_{\mathrm{K}}^{\,l}\,\tilde{s}_l^{\,\mathrm{K}} - m_{\mathrm{K}}c^2\right] \tag{III,12}$$

$$\partial_l\left(\frac{\partial\hat{\mathbf{\Phi}}_{\mathrm{K}}}{\partial q_l^{\mathrm{K}}}\right) = -\frac{\hbar^2}{m_{\mathrm{K}}}\partial_l\mathbf{q}_{\mathrm{K}}^{l} \tag{III,13}$$

Covariant wave equation:

$$\boxed{\hbar^2 q_{\mathrm{K};l}^{;l} = q_{\mathrm{K}}\left[\left(s_{\mathrm{K}}^{l} + \frac{e_{\mathrm{K}}}{c}a_{\overline{\mathrm{K}}}^{l}\right)\left(s_l^{\mathrm{K}} + \frac{e_{\mathrm{K}}}{c}a_l^{\overline{\mathrm{K}}}\right) - m_{\mathrm{K}}^2c^2\right]} \,. \tag{III,14}$$

The shape of particle K stays unchanged while $\dot{q}_{\mathrm{K}} = 0$, (III,15)

given a parameter of stationarity $\dot{s}_{\mathrm{K}} \overset{!}{=} constant\big|_t := -\varepsilon_{\mathrm{K}}$. (III,16)

The wave equation (III,14) for the shape scalar q_{K} implies in the formal limit $\hbar \to 0$ the conventional Hamilton-Jacobi equation for charged particles in an electromagnetic field.

III.3) The continuity equation for rest mass and charge by variation with respect to the microscopic action scalar *s*

$$\frac{\partial \widehat{\boldsymbol{\Phi}}_{\mathrm{K}}}{\partial s^{\mathrm{K}}} = 0 \,, \tag{III,17}$$

$$\partial_l \left\{ \frac{\partial \widehat{\boldsymbol{\Phi}}_{\mathrm{K}}}{\partial s_l^{\mathrm{K}}} \right\} = -\frac{1}{m_{\mathrm{K}}} \partial_l \left\{ q_{\mathrm{K}}^2 \left(\mathbf{s}_{\mathrm{K}}^{\,l} + \frac{e_{\mathrm{K}}}{c} \mathbf{a}_{\overline{\mathrm{K}}}^{\,l} \right) \right\} . \tag{III,18}$$

The covariant continuity equation

$$\boxed{\tilde{j}^{\,l}_{\mathrm{K};l} \equiv \frac{1}{\sqrt{g}} \partial_l \tilde{\mathbf{j}}^{\,l}_{\mathrm{K}} = 0} \tag{III,19}$$

with the natural normalization

$$-\frac{1}{m_{\mathrm{K}} c^2} \int q_{\mathrm{K}}^2 \left(\dot{\mathbf{s}}_{\mathrm{K}} + e_{\mathrm{K}} \mathbf{a}_{\overline{\mathrm{K}}}^{\,0} \right) \mathrm{d}V \stackrel{!}{=} 1 \tag{III,20}$$

yields

$$\int \tilde{\rho}_{\mathrm{K}} \mathrm{d}V \stackrel{!}{=} e_{\mathrm{K}} \equiv \pm e \,, \tag{III,21}$$

as well as with respect to (II,4)

$$\int \tilde{\mu}_{\mathrm{K}} \mathrm{d}V \stackrel{!}{=} m_{\mathrm{K}} \,. \tag{III,22}$$

III.4) The Einstein equations and a quantum energy-momentum tensor of field and matter by variation with respect to the fundamental tensor g_{ik}

A new treatment yields
$$\frac{\partial \mathbf{G}}{\partial g^{ik}} + g_{ia} g_{kb} \partial_l \left(\frac{\partial \mathbf{G}}{\partial g_{lab}} \right) = -2\kappa \frac{\partial \hat{\mathbf{\Phi}}}{\partial g^{ik}} \qquad \text{(III,23)}$$

resulting in
$$\mathbf{E}_{ik} \equiv \mathbf{R}_{ik} - \tfrac{1}{2} \mathbf{R} g_{ik} = \kappa \mathbf{T}_{ik} \quad , \qquad \text{(III,24)}$$

with the quantum tensor of matter
$$\mathbf{T}_{ik} \equiv \sum_{\mathrm{K}} \mathbf{T}_{ik}^{\mathrm{K}} \quad , \qquad \text{(III,25)}$$

where
$$\boxed{\mathbf{T}_{ik}^{\mathrm{K}} = -\tfrac{1}{2} \left(f_{im}^{\overline{\mathrm{K}}} \mathbf{f}_k^{\mathrm{K}m} + \mathbf{f}_{km}^{\overline{\mathrm{K}}} f_i^{\mathrm{K}m} \right) + \frac{1}{m_{\mathrm{K}}} q_{\mathrm{K}}^2 \tilde{s}_i^{\mathrm{K}} \tilde{\mathbf{s}}_k^{\mathrm{K}} + \frac{\hbar^2}{m_{\mathrm{K}}} q_i^{\mathrm{K}} \mathbf{q}_k^{\mathrm{K}} + \tfrac{1}{4} g_{ik} \left[f_{\mathrm{K}}^{lm} f_{lm}^{\overline{\mathrm{K}}} - \frac{\hbar^2}{m_{\mathrm{K}}} \left(q_{\mathrm{K}}^2 \right)_{;l}^{;l} \right]} , \qquad \text{(III,26)}$$

From (III,24) it follows explicitly
$$T_{i;k}^{k} \equiv \frac{1}{\sqrt{\mathbf{g}}} \left(\partial_k \mathbf{T}_i^k - \tfrac{1}{2} \mathbf{T}^{lm} g_{ilm} \right) = 0 \, , \qquad \text{(III,27)}$$

Which can be written in the form
$$\partial_k \mathbf{V}_i^k \equiv \partial_k \left(\mathbf{T}_i^k + \mathbf{t}_i^k \right) = 0 \, . \qquad \text{(III,28)}$$

Neglecting the fields f_{ik}^{K}, because of
$$\partial_k \left(\mu_{\mathrm{K}} \tilde{u}_{\mathrm{K}}^k \right) = 0 \, , \qquad \text{(III,29)}$$

(III,27) approximately means
$$\frac{d\tilde{u}_i^{\mathrm{K}}}{d\sigma} - \frac{1}{2} \tilde{u}_{\mathrm{K}}^k \tilde{u}_{\mathrm{K}}^l \partial_i g_{kl} = 0 \, . \qquad \text{(III,30)}$$

IV) Verification of the Klein-Gordon equation in its complex form, and the Schrödinger equation

A substitution in (III,13), (III,18) of
$$q_{\mathrm{K}} \equiv \sqrt{\psi_{\mathrm{K}}\psi_{\mathrm{K}}^{*}} \qquad \text{(IV,1)}$$

and
$$s_{\mathrm{K}} \equiv -\frac{\mathrm{i}\hbar}{2}\ln\left(\psi_{\mathrm{K}}/\psi_{\mathrm{K}}^{*}\right), \qquad \text{(IV,2)}$$

yields the Klein-Gordon
$$\boxed{\left(\mathrm{i}\hbar\partial_l - \frac{e_{\mathrm{K}}}{c}a_l^{\overline{\mathrm{K}}}\right)\left(\mathrm{i}\hbar\partial^l - \frac{e_{\mathrm{K}}}{c}\mathbf{a}^l_{\overline{\mathrm{K}}}\right)\psi_{\mathrm{K}} = m_{\mathrm{K}}^2c^2\Psi_{\mathrm{K}}} \qquad \text{(IV,3)}$$

equation in a covariant form for
$$\psi_{\mathrm{K}} \equiv q_{\mathrm{K}}\,\mathrm{e}^{\mathrm{i}s_{\mathrm{K}}/\hbar}. \qquad \text{(IV,4)}$$

Laue's theorem implies
$$\int\left|\psi_{\mathrm{K}\nu}^{(\text{stationary})}\right|^2\mathrm{d}V \overset{!}{=} \frac{\varepsilon_{\mathrm{K}\nu}}{m_{\mathrm{K}}c^2} \equiv 1 + \frac{\Delta\varepsilon_{\mathrm{K}\nu}}{m_{\mathrm{K}}c^2}. \qquad \text{(IV,5)}$$

A related substitution according to
$$\chi_{\mathrm{K}} \equiv Q_{\mathrm{K}}\,\mathrm{e}^{\mathrm{i}\sigma_{\mathrm{K}}/\hbar}, \qquad \text{(IV,6)}$$

where
$$s_{\mathrm{K}} \equiv -m_{\mathrm{K}}c^2t + \sigma_{\mathrm{K}}(t,\vec{r}), \qquad \text{(IV,7)}$$

yields approximately ($c \to \infty$) as far as
$$\frac{1}{c^2}\left|\dot{\sigma}_{\mathrm{K}} + e_{\mathrm{K}}\varphi_{\overline{\mathrm{K}}}\right| \ll m_{\mathrm{K}} \qquad \text{(IV,8)}$$

the Schrödinger equation
$$(\mathrm{i}\hbar\partial_t - e_{\mathrm{K}}\varphi_{\overline{\mathrm{K}}})\chi_{\mathrm{K}} - \frac{1}{2m_{\mathrm{K}}}\left(\mathrm{i}\hbar\vec{\nabla} + \frac{e_{\mathrm{K}}}{c}\vec{a}_{\overline{\mathrm{K}}}\right)^2\chi_{\mathrm{K}} = 0. \qquad \text{(IV,9)}$$

V) The consistency of the energy-momentum-stress tensor and its scalar, vector, and tensor parts

The natural parts of

$$T_{ik}^{\mathrm{L}} \equiv L_{ik}^{\mathrm{L}} + M_{ik}^{\mathrm{L}} + Q_{ik}^{\mathrm{L}} \tag{V,1}$$

are the tensor part

$$\boxed{L_{ik}^{\mathrm{L}} \equiv -f_{im}^{\;\bar{\mathrm{L}}} f_{k}^{\;\mathrm{L}m} + \tfrac{1}{4} g_{ik} f_{lm}^{\;\bar{\mathrm{L}}} f_{\mathrm{L}}^{\;lm}} \quad \{\text{field strengths}\}, \tag{V,2}$$

the vector part

$$\boxed{M_{ik}^{\mathrm{L}} \equiv \frac{1}{m_{\mathrm{L}}} q_{\mathrm{L}}^{2} \left(s_{i}^{\;\mathrm{L}} + \frac{e_{\mathrm{L}}}{c} a_{i}^{\;\bar{\mathrm{L}}} \right)\left(s_{k}^{\;\mathrm{L}} + \frac{e_{\mathrm{L}}}{c} a_{k}^{\;\bar{\mathrm{L}}} \right)} \quad \{\text{extended potentials}\}, \tag{V,3}$$

and the scalar part

$$\boxed{Q_{ik}^{\mathrm{L}} \equiv \frac{\hbar^{2}}{m_{\mathrm{L}}} \left[q_{i}^{\;\mathrm{L}} q_{k}^{\;\mathrm{L}} - \tfrac{1}{4} g_{ik} \left(q_{\mathrm{L}}^{2} \right)_{;l}^{;l} \right]} \quad \{\text{quantum shape scalar}\}. \tag{V,4}$$

A direct SRT-calculation

$$\partial_{k} T_{i\,\mathrm{L}}^{k} = \tfrac{1}{2} \left(f_{\mathrm{L}}^{\;km} \partial_{k} f_{im}^{\;\bar{\mathrm{L}}} - f_{\bar{\mathrm{L}}}^{\;km} \partial_{k} f_{im}^{\;\mathrm{L}} \right), \tag{V,5}$$

or at an alternative sorting

$$\partial_{k} T_{i\,\mathrm{K}}^{k} = \tfrac{1}{2} \left(f_{im}^{\;\bar{\mathrm{K}}} \tilde{j}_{\mathrm{K}}^{\;m} - f_{im}^{\;\mathrm{K}} \tilde{j}_{\bar{\mathrm{K}}}^{\;m} \right), \tag{V,6}$$

proves consistency in pairs of the whole:

$$\boxed{\partial_{k} \sum_{\mathrm{L/K}} T_{i\,\mathrm{L/K}}^{k} = 0}, \tag{V,7}$$

while

$$\partial_{k} \left(M_{i\,\mathrm{L}}^{k} + Q_{i\,\mathrm{L}}^{k} \right) = f_{ik}^{\;\bar{\mathrm{L}}} \tilde{j}_{\mathrm{L}}^{\;k}. \tag{V,8}$$

VI) Identification of the electron's energy with its parameter of stationarity by calculation from the energy-momentum tensor of the H-atom

Energy-momentum tensor H-atom:
$$T^{k}_{i\,(\mathrm{H})} = T^{k}_{i\,(\mathrm{e})} + T^{k}_{i\,(\mathrm{p})} \,. \tag{VI,1}$$

Presupposed potentials (neglecting spin):
$$a^{\,l}_{\mathrm{p}} = \left[\varphi_{\mathrm{p}}(\vec r),\, \vec 0\right], \qquad a^{\,l}_{\mathrm{e}} = \left[\varphi_{\mathrm{e}}(\vec r),\, \vec a_{\mathrm{e}}(\vec r)\right]. \tag{VI,2}$$

Stationary KG-solutions:
$$\psi_{(\mathrm{e,p})\,\nu} \equiv q_{(\mathrm{e,p})\,\nu}(\vec r)\, \mathrm{e}^{-\mathrm{i}\,\varepsilon_{(\mathrm{e,p})\,\nu} t/\hbar \,+\, \mathrm{i}\, s^{\mathrm{spatial}}_{(\mathrm{e,p})\,\nu}(\vec r)/\hbar} \,. \tag{VI,3}$$

The energy densities:
$$\boxed{\begin{aligned} T^{0\,(\mathrm{stationary})}_{0\,(\mathrm{e})} &= \frac{1}{2}\vec\nabla\varphi_{\mathrm{e}}\vec\nabla\varphi_{\mathrm{p}} + \frac{\tilde\rho_{\mathrm{e}}}{e_{\mathrm{e}}}\left(\varepsilon_{\mathrm{e}} - e_{\mathrm{e}}\varphi_{\mathrm{p}}\right) + \frac{1}{4}\frac{\hbar^2}{m_{\mathrm{e}}}\Delta\left(q_{\mathrm{e}}^2\right), \\ T^{0\,(\mathrm{stationary})}_{0\,(\mathrm{p})} &= \frac{1}{2}\vec\nabla\varphi_{\mathrm{e}}\vec\nabla\varphi_{\mathrm{p}} + \frac{\tilde\rho_{\mathrm{p}}}{e_{\mathrm{p}}}\left(\varepsilon_{\mathrm{p}} - e_{\mathrm{p}}\varphi_{\mathrm{e}}\right) + \frac{1}{4}\frac{\hbar^2}{m_{\mathrm{p}}}\Delta\left(q_{\mathrm{p}}^2\right). \end{aligned}} \tag{VI,4}$$

Energy H-atom:
$$E^{(\mathrm{stationary})}_{(\mathrm{H})} \equiv \int T^{0\,(\mathrm{stationary})}_{0\,(\mathrm{H})}\,\mathrm{d}V = \varepsilon_{\mathrm{e}}\int\frac{\tilde\rho_{\mathrm{e}}}{e_{\mathrm{e}}}\,\mathrm{d}V + \int\left[\frac{\tilde\rho_{\mathrm{p}}}{e_{\mathrm{p}}}\left(\varepsilon_{\mathrm{p}} - e_{\mathrm{p}}\varphi_{\mathrm{e}}\right)\right]\mathrm{d}V \,. \tag{VI,5}$$

Using $\varepsilon_{\mathrm{p}} \equiv m_{\mathrm{p}}c^2 + \Delta\varepsilon_{\mathrm{p}}$:
$$E^{(\mathrm{stationary})}_{(\mathrm{H})\infty} = \varepsilon_{\mathrm{e}} + m_{\mathrm{p}}c^2 + \lim_{m_{\mathrm{p}}\to\infty}\int\frac{\tilde\rho_{\mathrm{p}}}{e_{\mathrm{p}}}\left(\Delta\varepsilon_{\mathrm{p}} - e_{\mathrm{p}}\varphi_{\mathrm{e}}\right)\mathrm{d}V \,. \tag{VI,6}$$

Since $\lim\int ..\,\mathrm{d}V \to 0$:
$$\boxed{E^{(\mathrm{stationary})}_{(\mathrm{H})\infty} = \varepsilon_{\mathrm{e}\nu} + m_{\mathrm{p}}c^2}, \tag{VI,7}$$

where $\varepsilon_{\mathrm{e}\nu}$ has been a mere parameter of stationarity so far.

VII) The energy-frequency relation for the H-atom and a modified concept of electromagnetic waves

a temporary superposition is
$$\psi_{(\mathrm{e})} \equiv \psi_1 + \psi_2 \tag{VII,1}$$

with a charge density
$$\tilde{\rho}_{(\mathrm{e})} = \bar{\bar{\rho}}_{(\mathrm{e})} + \frac{-e}{m_{\mathrm{e}}c^2}\left[\varepsilon_{(\mathrm{e})1} + \varepsilon_{(\mathrm{e})2} + 2e\varphi_{(\mathrm{p})}\right] q_1 q_2 \cos\left[\omega_{12}t + \delta_{21}(\vec{r})\right], \tag{VII,2}$$

oscillating at the frequency
$$\boxed{\omega_{12} \equiv \frac{|\varepsilon_{12}|}{\hbar}}, \tag{VII,3}$$

where
$$\varepsilon_{12} = \varepsilon_{(\mathrm{e})1} - \varepsilon_{(\mathrm{e})2} \tag{VII,4}$$

is the energy difference, and
$$\delta_{21}(\vec{r}) \equiv \frac{1}{\hbar}\left[s_2^{\mathrm{spatial}}(\vec{r}) - s_1^{\mathrm{spatial}}(\vec{r})\right] \tag{VII,5}$$

The energy-momentum tensor
$$L^k_{i(\mathrm{H})} = -f^{\mathrm{p}}_{il} f_{\mathrm{e}}^{kl} - f^{\mathrm{e}}_{il} f_{\mathrm{p}}^{kl} + \tfrac{1}{2}\delta^k_i f_{\mathrm{e}}^{lm} f^{\mathrm{p}}_{lm} \tag{VII,6}$$

of the field, where in case of the H-atom
$$f^{\mathrm{p}}_{ik} \approx \frac{m_{\mathrm{e}}}{m_{\mathrm{p}}} f^{\mathrm{e}}_{ik}, \tag{VII,7}$$

may be converted using
$$f_{ik} :\approx \sqrt{\frac{2m_{\mathrm{e}}}{m_{\mathrm{p}}}} f^{\mathrm{e}}_{ik} \approx \sqrt{\frac{2m_{\mathrm{p}}}{m_{\mathrm{e}}}} f^{\mathrm{p}}_{ik} \tag{VII,8}$$

to find in usual notation
$$L^k_{i(\mathrm{H})} = -f_{il} f^{kl} + \tfrac{1}{4}\delta^k_i f_{lm} f^{lm}. \tag{VII,9}$$

Appendix: Steps to a consistent variational principle of matter and field – and a special historical remark

Larmor (1900):
$$S = \int \left[\tfrac{1}{4} F^{lm} F_{lm} + A_l j^l \right] \mathrm{d}\Omega \,, \tag{A,1}$$

Hilbert (1915):
$$S = \int \left[\tfrac{1}{4} \mathbf{F}^{lm} F_{lm} + A_l \mathbf{j}^l + \tfrac{1}{\kappa} \mathbf{R} \right] \mathrm{d}\Omega \,, \tag{A,2}$$

Weyl/a (1918):
$$S = \int \left[\tfrac{1}{4} \mathbf{F}^{lm} F_{lm} + A_l \mathbf{j}^l + \mu_0 c^2 + \tfrac{1}{\kappa} \mathbf{G} \right] \mathrm{d}\Omega \,, \tag{A,3}$$

Weyl/b (1918):
$$S = \tfrac{1}{4} \int \mathbf{F}^{ik} F_{ik} \,\mathrm{d}\Omega + \sum \tfrac{e_0}{c} \int A_i \,\mathrm{d}x^i + \sum m_0 c \int \mathrm{d}s + \tfrac{1}{\kappa} \int \mathbf{G} \,\mathrm{d}\Omega \,, \tag{A,4}$$

I found:
$$S = \int \left[\sum_{\mathrm{K}} \left(\tfrac{1}{4} \mathbf{f}_{\mathrm{K}}^{lm} f_{lm}^{\overline{\mathrm{K}}} + \tfrac{1}{2} \tilde{a}_l^{\overline{\mathrm{K}}} \tilde{\mathbf{j}}_{\mathrm{K}}^l + \tfrac{1}{2} \mu_{\mathrm{K}} c^2 - \frac{\hbar^2}{2 m_{\mathrm{K}}} \mathbf{q}_{\mathrm{K}}^l q_l^{\mathrm{K}} \right) + \tfrac{1}{2\kappa} \mathbf{G} \right] \mathrm{d}\Omega \,. \tag{A,5}$$

...

The phenomenological energy-momentum tensor of matter used and discussed by Albert Einstein and his friend Marcel Grossmann
in their fundamental work of 1913 is approximately derived here in the limit $\hbar \to 0$ from (V,1), (V,3) to yield

$$T_{ik}^{(\mathrm{dust})} \approx M_{ik}^{(\mathrm{dust})} \equiv \sum_{\mathrm{L}} \frac{1}{m_{\mathrm{L}}} q_{\mathrm{L}}^2 \tilde{s}_i^{\,\mathrm{L(dust)}} \tilde{s}_k^{\,\mathrm{L(dust)}} :\approx \sum_{\mathrm{L}} \mu_{\mathrm{L}} c^2 U_i^{\mathrm{L}} U_k^{\mathrm{L}} \,. \tag{A,6}$$

Comments to the sheets
using the numbering above:

I) The energy-momentum postulate (I,1) requires within SRT – i.e. within freely falling inertial systems – the conservation laws (I,2).

Though flying temporarily in the same direction, both objects together with their respective carrier of interaction have to fulfill (I,2) with respect to S and S'.

Since macroscopic objects are composed of charged particles, each process is subject to electrodynamics.

Insertion into (I,2) of the conventional energy-momentum tensor (I,3) – phenomenologically combined of two independent parts so far – yields the electrodynamic equations of motion (I,4).

To solve the paradoxa according to (I,2), the components of T_i^k must fulfill the standard conditions of continuity and integrability.

II) A scalar Q is defined in relation (II,4) which with respect to a single particle obviously determines its shape. This shape scalar will play a major role.

Obviously, relation (II,7) would mean an unacceptable consequence if there was not the well-known gauge invariance. On the other hand, however, the same gauge invariance might have blocked the view in that it seemed to prove the electromagnetic potentials to be of no physical relevance.

But now, solution (II,9) will be the key to a consistent variational principle in that it is fixing a necessary relation between current density and the electromagnetic potentials which seems to have been ignored within conventional electrodynamics so far.

III) The action density assigned to a single particle K (III,2) includes a familiar looking scalar of the electromagnetic field which, however, means field products of different particles only. This prevents from any self energy or renormalization problems – though: the actual reason for this approach is the impossibility, to get otherwise a consistent energy-momentum tensor satisfying (I,2).

The second part simply means a product of 4-dimensional current density and electromagnetic potentials, thereby using the key found in (II,9).

The third part is obviously representing the rest-mass density, though of the bound rest mass according to (IV,5), whereas the corresponding free rest mass density of a particle K is used in (III,22).

The last part is added as the simplest expression leading to an equation describing the behavior of the shape scalar. If otherwise its partial derivatives would be completely missing, such an equation would be missing, too.

Temporarily, $\hbar$ means nothing but an unknown constant.

III.1) In the 2. pair of the quantized Maxwell equations there does not appear the electromagnetic potential of the respective particle K, but the potentials of all other particles $\overline{\mathrm{K}}$ instead.

III.2) Here is found a real inhomogeneous wave equation for the shape scalar q_{K} in a covariant form. In the formal limit $\hbar \to 0$, this wave equation implies the conventional Hamilton-Jacobi equation for charged particles in an external electromagnetic field.

The relations (III,15), (III,16) obviously define possible stationary states of particles, given that the corresponding solutions really exist.

III.3) The covariant continuity equation (III,19) holds for the rest-mass density, too, since this is presupposed to be directly proportional to the charge density in (II,4).

It is important that such an equation also results from the 2. pair of the quantized Maxwell equations before.

III.4) The derivation of Einstein's equations resulting in (III,24-26) means a completion by a consistent energy-momentum tensor of matter, which is not only added phenomenologically as before. Its compatibility will be shown in (A,6) of the Appendix.

Therefore, the contracted Bianchi identities here imply Einstein's gravitational equations of motion (III,30) in the formal limit $\hbar \to 0$ approximately.

IV) Though it is easy to verify the complex Klein-Gordon equation using the explicit substitutions (IV,1), (IV,2) it might have taken more time to derive it from (III,13), (III,18) if history had gone another way – not to mention any consequences.

The complex quantum scalar ψ of the Klein-Gordon equation is shown in (IV,5) to require a new normalization according to (III,21-22), which solely proves consistent with conservation of charge and the energy frequency relation at last.

V) The conservation laws (I,2) are always fulfilled in pairs for particles which are contributing to the whole energy-momentum tensor T_i^k.

They are approximately fulfilled for single particles, too, as long as those are moving in source-free regions of electromagnetic fields only.

The scalar part (V,4) of the energy-momentum tensor contains contributions due to the particle's shape. It are these contributions which – based on the conventional assumption of fictive point particles – are commonly associated to Heisenbergs uncertainty or the zero-point energy.

VI) The characteristic result (VI,7) – where that otherwise mysterious parameter of stationarity $\varepsilon_{e\nu}$ is straightforwardly calculated to be the energy of a bound particle K – is given here as an approximation at the example of the H-atom only.

As it can be seen from (VI,5), (VI,6), this calculation would not have worked using the conventional normalization of ψ instead of (III,21).

VII) Now the energy-frequency relation is easy to understand following an argumentation of Schroedinger where, however, any derivation of the fundamental energy-parameter identity in the sense of (VI,7) seemed impossible at that time.

The concept of electromagnetic waves has to be modified here in that
a) the energy quantum of radiation is determined by its frequency and the other way round; b) a single field does not carry any energy or momentum.

At the H-atom for example, only the superposed fields of electron and proton will carry energy and momentum, since during any transitions between different stationary states their charge distributions are oscillating approximately in phase.

Notation

Besides the well-known Landau-Lifshitz Notation or any self-explanatory abbreviations, only a few special symbols may be listed here:

Some italic capital letters like in A_i or the 4-velocity U^i will denote macroscopically smoothed quantities in contrast to their microscopic counterparts like in $a_i^{\overline{\mathrm{K}}}$ or $u^i{}_{\mathrm{K}}$.

S, s – the action scalar, where e. g. $s_i \equiv \partial_i s \equiv \partial s / \partial x^i$
Q, q – the shape scalar, where $q_l \equiv \partial_l q \equiv \partial q / \partial x^l$
A_i, a_i – the electrodynamic potentials, where $A_{ik} \equiv \partial_i A_k \equiv \partial A_k / \partial x^i$
g_{ik} – the fundamental gravitational tensor where e. g. $g_{lik} \equiv \partial_l g_{ik} \equiv \partial g_{ik} / \partial x^l$
$\sqrt{g}$ – the square root of its negative determinant g, where any $\mathbf{X} \equiv \sqrt{g}\, X$
$\mathrm{d}\sigma$ – the line element of general relativity theory [though, σ_{K} s. (IV,7)]
$c\,\mathrm{d}\Omega \equiv c\,\mathrm{d}t\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

A tilde like in the extended macroscopically smoothed 4-potentials $\tilde{A}_i \equiv A_i + c/e_0 \cdot \partial_i S$ where $A^k{}_{;k} = 0$, or in $\tilde{s}_i^{\mathrm{K}} \equiv s_i^{\mathrm{K}} + e_{\mathrm{K}}/c \cdot a_i^{\overline{\mathrm{K}}}$ indicates a combination of differentiable components.

A, B, C ... (non-italic) are explicit particle indices, whereas K, L indicate different combinations of summands in symbolic sums. In contrast, the indices $\overline{\mathrm{K}}, \overline{\mathrm{L}}$ imply an immediate summation over all particles except for K, L.

$m_{\mathrm{K}}, e_{\mathrm{K}}, \mu_{\mathrm{K}}, \rho_{\mathrm{K}} \equiv m_0, e_0, \mu_0, \rho_0$ of particle K.

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You will find some more necessary aspects – and the detailed notation, too – within approx. 100 pages of "Skizze einer offenen relativistischen Theorie von Elektrodynamik, Gravitation, Quantenmechanik" in August 2006 sGw. A link will be placed on my website). – In all cases of doubt which seem almost unavoidable because of the limited space and time of this talk as well as of the limited eloquence of the speaker, please use the e-print just mentioned as the valid expression of my approach.*

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*) Additional note of 21 December 2008: Another version of this talk has been published in „PROCEEDINGS OF THE MG11 MEETING ON GENERAL RELATIVITY“, World Scientific, Singapore)